Session 5

Defining Types

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Data Declarations

A new type can be defined by specifying its set of values using a data declaration.

```
data Bool = False | True
```

Bool is a new type, with two new values False and True.

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Values of new types can be used in the same ways as those of built-in types. For example, given

```
data Answer = Yes | No | Unknown
```

we can define:

```
answers        :: [Answer]
answers         = [Yes,No,Unknown]

flip           :: Answer → Answer
flip Yes        = No
flip No         = Yes
flip Unknown    = Unknown
```

Functie zonder argumenten die een lijst van Answer data teruggeeft

Def. Vd functie: geef constante lijst van alle 3 de antwoorden terug Indien gewenst zou je er dubbelen in kunnen steken maar dat is hier natuurlijk ongewenst...

2de functie die het Answer type gebruikt op een meer zinvolle manier

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The constructors in a data declaration can also have parameters. For example, given

```
data Shape = Circle Float
           | Rect Float Float
```

Ctor pars

we can define:

```
square          :: Float → Shape
square n         = Rect n n

area            :: Shape → Float
area (Circle r) = pi * r^2
area (Rect x y) = x * y
```

Ctor call met argn

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Similarly, data declarations themselves can also have parameters. For example, given

```
data Maybe a = Nothing | Just a
```

Ctor met 1 parameter

we can define:

```
return          :: a → Maybe a
return x         = Just x

(>>=) :: Maybe a → (a → Maybe b) → Maybe b
Nothing >>= _  = Nothing
Just x  >>= f = f x
```

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Recursive Types

In Haskell, new types can be defined in terms of themselves. That is, types can be recursive.

```
data Nat = Zero | Succ Nat
```

Nat is a new type, with constructors Zero :: Nat and Succ :: Nat $\rightarrow$ Nat.

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Note:

- A value of type Nat is either Zero, or of the form Succ n where n :: Nat. That is, Nat contains the following infinite sequence of values:

```
Zero

Succ Zero

Succ (Succ Zero)

⋮
```

- We can think of values of type Nat as natural numbers, where Zero represents 0, and Succ represents the successor function (1 +).

- For example, the value

```
Succ (Succ (Succ Zero))
```

represents the natural number

```
1 + (1 + (1 + 0))  =  3
```

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Using recursion, it is easy to define functions that convert between values of type Nat and Int:

```
nat2int           :: Nat → Int
nat2int Zero       = 0
nat2int (Succ n) = 1 + nat2int n


int2nat           :: Int → Nat
int2nat 0          = Zero
int2nat (n+1)      = Succ (int2nat n)
```

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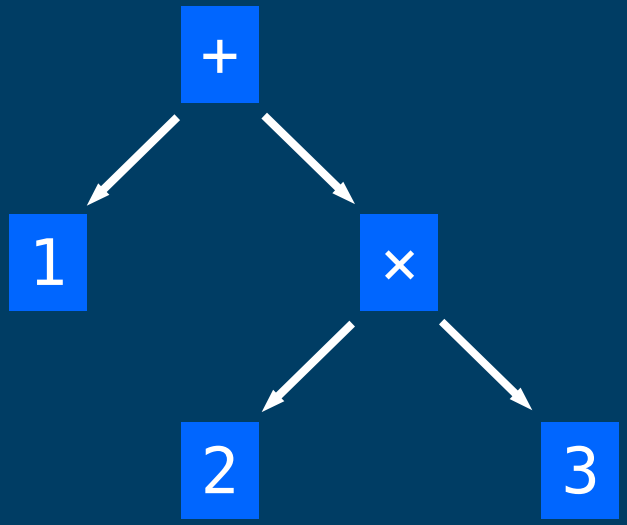
Arithmetic Expressions

Consider a simple form of expressions built up from integers using addition and multiplication.



Using recursion, a suitable new type to represent such expressions can be defined by:

```
data Expr = Val Int
          | Add Expr Expr
          | Mul Expr Expr
```

For example, the expression on the previous slide would be represented as follows:

```
Add (Val 1) (Mul (Val 2) (Val 3))
```

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Using recursion, it is now easy to define functions that process expressions. For example:

```
size          :: Expr → Int
size (Val n)   = 1
size (Add x y) = size x + size y
size (Mul x y) = size x + size y

eval          :: Expr → Int
eval (Val n)   = n
eval (Add x y) = eval x + eval y
eval (Mul x y) = eval x * eval y
```

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Exercises

(1) Define an add function, once using conversion functions (int2nat, ...) and once using recursion.

(2) Define a type BinaryTree, with nodes and leafs (nodes have 2 subtrees, leafs don't). Each node/leaf has a value. Define a function “occurs”, which checks if a given value occurs in a tree.

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